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Pearson Edexcel
Level 3 GCE

Centre Number

Candidate Number

Further Mathematics

Advanced
Further Mathematics Option 1
Paper 3: Further Mechanics 1
Further Mathematics Option 2
Paper 4: Further Mechanics 1

Sample Assessment Material for first teaching September 2017

Time: 1 hour 30 minutes

Paper Reference
9FM0/3C
9FM0/4C

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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Answer ALL questions. Write your answers in the spaces provided.

Unless otherwise indicated, whenever a numerical value of g is required, take $g = 9.8\,\text{m}\,\text{s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

1. A particle P of mass 0.5 kg is moving with velocity $(4\mathbf{i} + \mathbf{j})\,\text{m}\,\text{s}^{-1}$ when it receives an impulse $(2\mathbf{i} - \mathbf{j})\,\text{N}\,\text{s}$.

Show that the kinetic energy gained by P as a result of the impulse is 12 J.

(6)

Question 1 continued

(Total for Question 1 is 6 marks)

2. A parcel of mass 5 kg is projected with speed $8\,\text{m}\,\text{s}^{-1}$ up a line of greatest slope of a fixed rough inclined ramp.

The ramp is inclined at angle α to the horizontal, where $\sin\alpha = \frac{1}{7}$

The parcel is projected from the point A on the ramp and comes to instantaneous rest at the point B on the ramp, where $AB = 14\,\text{m}$.

The coefficient of friction between the parcel and the ramp is μ.

In a model of the parcel's motion, the parcel is treated as a particle.

(a) Use the work-energy principle to find the value of μ.

(5)

(b) Suggest one way in which the model could be refined to make it more realistic.

(1)

Question 2 continued

(Total for Question 2 is 6 marks)

3. A particle of mass m kg lies on a smooth horizontal surface.

Initially the particle is at rest at a point O between two fixed parallel vertical walls.

The point O is equidistant from the two walls and the walls are 4 m apart.

At time $t = 0$ the particle is projected from O with speed u m s^{-1} in a direction perpendicular to the walls.

The coefficient of restitution between the particle and each wall is $\frac{3}{4}$

The magnitude of the impulse on the particle due to the first impact with a wall is λmu N s.

(a) Find the value of λ.

(3)

The particle returns to O, having bounced off each wall once, at time $t = 7$ seconds.

(b) Find the value of u.

(5)

Question 3 continued

(Total for Question 3 is 8 marks)

4.

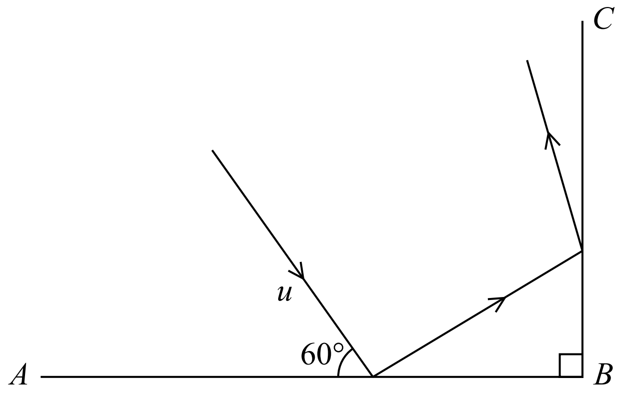

Figure 1

Figure 1 represents the plan view of part of a horizontal floor, where *AB* and *BC* are perpendicular vertical walls.

The floor and the walls are modelled as smooth.

A ball is projected along the floor towards *AB* with speed $u\,\text{m}\,\text{s}^{-1}$ on a path at an angle of 60° to *AB*. The ball hits *AB* and then hits *BC*.

The ball is modelled as a particle.

The coefficient of restitution between the ball and wall *AB* is $\frac{1}{\sqrt{3}}$

The coefficient of restitution between the ball and wall *BC* is $\sqrt{\frac{2}{5}}$

(a) Show that, using this model, the final kinetic energy of the ball is 35% of the initial kinetic energy of the ball.

(8)

(b) In reality the floor and the walls may not be smooth. What effect will the model have had on the calculation of the percentage of kinetic energy remaining?

(1)

Question 4 continued

Question 4 continued

Question 4 continued

(Total for Question 4 is 9 marks)

5. A car of mass 600 kg is moving along a straight horizontal road.

At the instant when the speed of the car is $v\,\text{m}\,\text{s}^{-1}$, the resistance to the motion of the car is modelled as a force of magnitude $(200 + 2v)$ N.

The engine of the car is working at a constant rate of 12 kW.

(a) Find the acceleration of the car at the instant when $v = 20$

(4)

Later on the car is moving up a straight road inclined at an angle θ to the horizontal,

where $\sin\theta = \dfrac{1}{14}$

At the instant when the speed of the car is $v\,\text{m}\,\text{s}^{-1}$, the resistance to the motion of the car from non-gravitational forces is modelled as a force of magnitude $(200 + 2v)$ N.

The engine is again working at a constant rate of 12 kW.

At the instant when the car has speed $w\,\text{m}\,\text{s}^{-1}$, the car is decelerating at $0.05\,\text{m}\,\text{s}^{-2}$.

(b) Find the value of w.

(5)

Question 5 continued

(Total for Question 5 is 9 marks)

6. [In this question **i** and **j** are perpendicular unit vectors in a horizontal plane.]

A smooth uniform sphere A has mass $2m$ kg and another smooth uniform sphere B, with the same radius as A, has mass $3m$ kg.

The spheres are moving on a smooth horizontal plane when they collide obliquely.

Immediately before the collision the velocity of A is $(3\mathbf{i} + 3\mathbf{j})\,\text{m s}^{-1}$ and the velocity of B is $(-5\mathbf{i} + 2\mathbf{j})\,\text{m s}^{-1}$.

At the instant of collision, the line joining the centres of the spheres is parallel to **i**.

The coefficient of restitution between the spheres is $\frac{1}{4}$

(a) Find the velocity of B immediately after the collision.

(7)

(b) Find, to the nearest degree, the size of the angle through which the direction of motion of B is deflected as a result of the collision.

(2)

Question 6 continued

Question 6 continued

Question 6 continued

(Total for Question 6 is 9 marks)

7. A particle P of mass m is attached to one end of a light elastic string of natural length a and modulus of elasticity $3mg$.

The other end of the string is attached to a fixed point O on a ceiling.

The particle hangs freely in equilibrium at a distance d vertically below O.

(a) Show that $d = \frac{4}{3}a$.

(3)

The point A is vertically below O such that $OA = 2a$.

The particle is held at rest at A, then released and first comes to instantaneous rest at the point B.

(b) Find, in terms of g, the acceleration of P immediately after it is released from rest.

(3)

(c) Find, in terms of g and a, the maximum speed attained by P as it moves from A to B.

(5)

(d) Find, in terms of a, the distance OB.

(3)

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DO NOT WRITE IN THIS AREA

Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 14 marks)

8. A particle P of mass $2m$ and a particle Q of mass $5m$ are moving along the same straight line on a smooth horizontal plane.

They are moving in opposite directions towards each other and collide directly.

Immediately before the collision the speed of P is $2u$ and the speed of Q is u.

The direction of motion of Q is reversed by the collision.

The coefficient of restitution between P and Q is e.

(a) Find the range of possible values of e.

(8)

Given that $e = \frac{1}{3}$

(b) show that the kinetic energy lost in the collision is $\frac{40mu^2}{7}$.

(5)

(c) Without doing any further calculation, state how the amount of kinetic energy lost in the collision would change if $e > \frac{1}{3}$

(1)

Question 8 continued

Question 8 continued

Question 8 continued

(Total for Question 8 is 14 marks)

TOTAL FOR PAPER IS 75 MARKS

Paper 3C/4C: Further Mechanics 1 Mark Scheme

Question	Scheme	Marks	AOs
1	Use Impulse-momentum principle	M1	2.1
	$2\mathbf{i}-\mathbf{j}=0.5\mathbf{v}-0.5(4\mathbf{i}+\mathbf{j})$	A1	1.1b
	$\frac{1}{2}\mathbf{v}=4\mathbf{i}-\frac{1}{2}\mathbf{j}$, $\quad \mathbf{v}=8\mathbf{i}-\mathbf{j}$ (m s^{-1})	A1	1.1b
	Use of $\text{KE}=\frac{1}{2}m\lvert\mathbf{v}\rvert^2-\frac{1}{2}m\lvert\mathbf{u}\rvert^2$	M1	2.1
	$=\frac{1}{2}\times0.5\times\{(64+1)-(16+1)\}$	A1	1.1b
	$=\frac{1}{4}\times48=12$ (J) $\quad$ *	A1*	1.1b
		(6)	
			(6 marks)

Notes:

M1: Difference of terms & dimensionally correct
A1: Correct unsimplified equation
A1: cao
M1: Must be a difference of two terms
Must be dimensionally correct
A1: Correct unsimplified equation
A1*: Complete justification of given answer

Question	Scheme	Marks	AOs
2(a)	$R = 5g\cos\alpha \left(= 5g \times \dfrac{4\sqrt{3}}{7} = 48.497... \right)$	M1	3.4
	Force due to friction = $\mu \times 5g\cos\alpha$	M1	3.4
	Work-Energy equation	M1	3.4
	$\frac{1}{2} \times 5 \times 64 = 5 \times 9.8 \times 14\sin\alpha + 14\mu R$	A1	1.1b
	$\mu = 0.0913$ or 0.091	A1	1.1b
		(5)	
(b)	Appropriate refinement	B1	3.5c
		(1)	
		(6 marks)	

Notes:

(a)
M1: Condone sin/cos confusion
M1: Use of $\mu \times$ their R
M1: Must be using work-energy. Requires all terms
Condone sin/cos confusion, sign errors and their R
A1: Correct in θ and μR
A1: Accept 0.0913 **or** 0.091

(b)
B1: e.g.
- do not model the parcel as a particle and therefore take air resistance into account
- take into account the dimensions/uniformity of the parcel

Question	Scheme	Marks	AOs
3(a)	Use NEL to find the speed of particle after the first impact $= eu = \frac{3}{4}u\frac{\pi}{2}$	B1	3.4
	Impulse $= \lambda mu = mv - mu = \pm\left[\frac{3}{4}mu - (-mu)\right]$	M1	3.1b
	$\lambda = \frac{7}{4}$	A1	1.1b
		(3)	
(b)	Use NEL to find the speed of the particle after the second impact $= \frac{3}{4}\times\frac{3}{4}u = \frac{9}{16}u$	B1	3.4
	Use of $s = vt$ to find total time	M1	3.1b
	$7 = \frac{2}{u} + \frac{4}{\frac{3}{4}u} + \frac{2}{\frac{9}{16}u}\left(= \frac{2}{u} + \frac{16}{3u} + \frac{32}{9u}\right)$	A1	1.1b
	Solve for *u*: $63u = 18 + 48 + 32$	M1	1.1b
	$u = \frac{98}{63} = \frac{14}{9}\left(= 1.\dot{5}\right)$	A1	1.1b
		(5)	
			(8 marks)

Notes:

(a)

B1: Using Newton's experimental law as a model to find the speed after the first impact

M1: Must be a difference of two terms, taking account of the change in direction of motion

A1: cao

(b)

B1: Using NEL as a model to find the speed after the second impact

M1: Needs to be used for at least one stage of the journey

A1: Ur equivalent

M1: Solve their linear equation for *u*

A1: Accept 1.56 or better

Question	Scheme	Marks	AOs
4(a)	Complete strategy to find the kinetic energy after the second impact	M1	3.1b
	Parallel to AB after collision: $u\cos 60°$	M1	3.1b
	Perpendicular to AB after collision: $\frac{1}{\sqrt{3}}u\sin 60°$	M1	3.4
	Components of velocity after first impact: $\frac{u}{2}, \frac{u}{2}$	A1	1.1b
	Parallel to BC after collision: $\frac{u}{2}$ $\left(u\times\frac{1}{\sqrt{3}}\sin 60°\right)$	M1	3.1b
	Perpendicular to BC after collision: $\sqrt{\frac{2}{5}}\times\frac{u}{2}\left(=\frac{1}{\sqrt{10}}u\right)$ $\left(\sqrt{\frac{2}{5}}\times u\cos 60°\right)$	M1	3.4
	Components of velocity after second impact: $\frac{u}{2}, \frac{u}{\sqrt{10}}$	A1	1.1b
	Final KE $=\frac{1}{2}m\left(\frac{u^2}{4}+\frac{u^2}{10}\right)$ $\left(=\frac{mu^2}{2}\times\frac{7}{20}\right)$		
	Fraction of initial KE $=\dfrac{\frac{mu^2}{2}\times\frac{7}{20}}{\frac{mu^2}{2}}=\frac{7}{20}=35\%$ *	A1*	2.2a
		(8)	
(b)	The answer is too large - rough surface means resistance so final speed will be lower	B1	3.5a
		(1)	
		(9 marks)	

Notes:

(a)

M1: Use of CLM parallel to the wall. Condone sin/cos confusion

M1: Use NEL as a model to find the speed perpendicular to the wall. Condone sin/cos confusion

A1: Both components correct with trig substituted (seen or implied)

M1: Use of CLM parallel to the wall. Condone sin/cos confusion

M1: Use NEL as a model to find the speed perpendicular to the wall. Condone sin/cos confusion

A1: Both components correct with trig substituted (seen or implied)

M1: Correct expression for total KE using their components after 2nd collision

A1*: Obtain **given answer** with sufficient working to justify it

(b)

B1: Clear explanation of how the modelling assumption has affected the outcome

Question	Scheme	Marks	AOs
5(a)	Use of $P = Fv$: $F = \frac{12000}{20}$	B1	3.3
	Equation of motion: $F - (200 + 2v) = 600a$	M1	3.4
	$600 - 240 = 600a$	A1ft	1.1b
	$360 = 600a$, $a = 0.6$ (m s^{-2})	A1	1.1b
		(4)	
(b)	Equation of motion:	M1	3.3
	$\frac{12000}{w} - (200 + 2w) - 600g\sin\theta = -600 \times 0.05$	A1 A1	1.1b 1.1b
	3 term quadratic and solve: $2w^2 + 590w - 12000 = 0$	M1	1.1b
	$w = \frac{-590 + \sqrt{590^2 + 96000}}{4} = 19.1$ (m s^{-1})	A1	1.1b
		(5)	
			(9 marks)

Notes:

(a)

B1: 600 or equivalent

M1: Use the model to form the equation of motion
Must include all terms .Condone sign errors

A1ft: Correct for their F

A1: cao

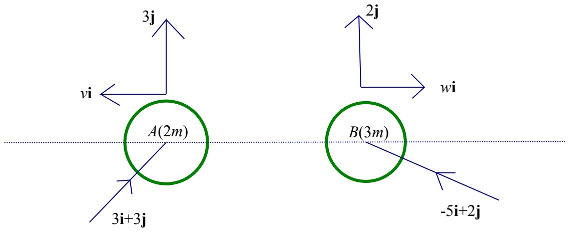
(b)

M1: Use the model to form the equation of motion
All terms needed. Condone sign errors and sin/cos confusion

A1: All correct A1A1
One error A1A0

M1: Dependent on the preceding M1. Use the equation of motion to form a 3-term quadratic in w only

A1: Accept 19. Do not accept more than 3 s.f.

Question	Scheme	Marks	AOs
6(a)			
	Overall strategy to find $\mathbf{v}_A$	M1	3.1a
	Velocity of A perpendicular to loc after collision $= 3\mathbf{j}$ (m s^{-1})	B1	3.4
	CLM parallel to loc	M1	3.1a
	$2m \times 3 - 3m \times 5 = 3mw - 2mv \quad (-9 = 3w - 2v)$	A1	1.1b
	Correct use of impact law	M1	3.1a
	$v + w = \frac{1}{4}(3+5) \quad (=2)$	A1	1.1b
	Solve for w $\quad 3w - 2v = -9$ $2v + 2w = 4$		
	$\mathbf{v}_B = -\mathbf{i} + 2\mathbf{j}$ (m s^{-1}),	A1ft	1.1b
		(7)	
(b)	$\cos\theta = \dfrac{(-5\mathbf{i}+2\mathbf{j}).(-\mathbf{i}+2\mathbf{j})}{\sqrt{29}\sqrt{5}}$	M1	3.1a
	$\theta = 41.63...° = 42°$ (nearest degree)	A1	1.1b
	Alternative method: $\tan^{-1} 2 - \tan^{-1}\frac{2}{5} = 41.63...° = 42°$ (nearest degree)		
		(2)	
		(9 marks)	

Notes:

(a)

M1: Correct overall strategy to form sufficient equations and solve for $\mathbf{v}_A$

B1: Use the model to find the component of $\mathbf{v}_A$ perpendicular to the line of centres

M1: Use CLM to form equation in v and w. Need all 4 terms, dimensionally correct

A1: Correct unsimplified

M1: Must be used the right way round

A1: Correct unsimplified

A1ft: $\mathbf{v}_B$ correct. Follow their $2\mathbf{j}$

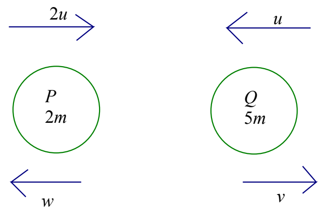
(b)

M1: Complete method for finding the required angle. Follow their $\mathbf{v}_B$

A1: cao

Question	Scheme	Marks	AOs
7(a)	In equilibrium $\Rightarrow$ no resultant vertical force	M1	2.1
	$\frac{3mgx}{a} = mg$	A1	1.1b
	$x = \frac{a}{3}$, $d = \frac{4}{3}a$ *	A1*	2.2a
		(3)	
(b)	Equation of motion:	M1	3.1a
	$\frac{3mga}{a} - mg = m\ddot{x}$	A1	1.1b
	$\ddot{x} = 2g$	A1	1.1b
		(3)	
(c)	Max speed at equilibrium position	B1	3.1a
	Work energy & use of EPE $= \frac{\lambda x^2}{2a}$	M1	3.1a
	$\frac{3mga^2}{2a} = \frac{3mg\left(\frac{a}{3}\right)^2}{2a} + \frac{1}{2}mv^2 + mg\frac{2a}{3}$	A1 A1	1.1b 1.1b
	$\frac{1}{2}v^2 = ga\left(\frac{3}{2} - \frac{1}{6} - \frac{2}{3}\right) = \frac{2}{3}ga$, $v = \sqrt{\frac{4ga}{3}}$	A1	1.1b
		(5)	
(d)	At max ht. KE = 0. EPE lost = GPE gained	M1	3.1a
	$\frac{3mga^2}{2a} = mgh$	A1	1.1b
	$OB = \frac{a}{2}$	A1	1.1b
		(3)	
			(14 marks)

Question 7 notes:	
(a)	
M1:	Use $T = \frac{\lambda x}{a}$ to form equation for equilibrium
A1:	Correct unsimplified equation
A1*:	Requires sufficient working to justify given answer plus a 'statement' that the required result has been achieved
(b)	
M1:	Use $T = \frac{\lambda x}{a}$ to form equation of motion Need all 3 terms. Condone sign errors
A1:	Correct unsimplified equation
A1:	cao
(c)	
B1:	Seen or implied
M1:	Form work-energy equation. All 4 terms needed Condone sign errors
A1:	Correct unsimplified equation A1A1 One error in the equation A1A0
A1:	cao
(d)	
M1:	Form energy equation
A1:	Correct unsimplified equation
A1:	cao

Question	Scheme	Marks	AOs
8(a)			
	Complete overall strategy to find v	M1	3.1a
	Use of CLM	M1	3.1a
	$2m\times 2u - 5m\times u = 5m\times v - 2m\times w$, $(-u = 5v - 2w)$	A1	1.1b
	Use of Impact law:	M1	3.1a
	$v + w = e(2u + u)$	A1	1.1b
	Solve for v: $-u = 5v - 2w$ $6eu = 2v + 2w$		
	$7v = u(6e - 1)$ $\left(v = \frac{u}{7}(6e-1)\right)$	A1	1.1b
	Direction of Q reversed: $v > 0$	M1	3.4
	$\Rightarrow 1 \geq e > \frac{1}{6}$	A1	1.1b
		(8)	
(b)	$e = \frac{1}{3} \Rightarrow v = \frac{u}{7}$, $w = \frac{6u}{7}$	B1	2.1
	Equation for KE lost	M1	2.1
	$\frac{1}{2}\times 2m\left(4u^2 - \frac{36u^2}{49}\right) + \frac{1}{2}\times 5m\left(u^2 - \frac{u^2}{49}\right)$	A1 A1	1.1b 1.1b
	$\frac{1}{2}mu^2\left(8 - \frac{72}{49} + 5 - \frac{5}{49}\right) = \frac{40mu^2}{7}$ *	A1*	2.2a
		(5)	
(c)	Increase $e \Rightarrow$ more elastic $\Rightarrow$less energy lost	B1	2.2a
		(1)	
			(14 marks)

Question 8 notes:	
(a)	
M1:	Complete strategy to form sufficient equations in v and w and solve for v
M1:	Use CLM to form equation in v and w Needs all 4 terms & dimensionally correct
A1:	Correct unsimplified equation
M1:	Use NEL as a model to form a second equation in v and w. Must be used the right way round
A1:	Correct unsimplified equation
A1:	for v or $7v$ correct
M1:	Use the model to form a correct inequality for their v
A1:	Both limits required
(b)	
B1:	Or equivalent statements
M1:	Terms of correct structure combined correctly
A1:	Fully correct unsimplified A1A1 One error on unsimplified expression A1A0
A1*:	cso. plus a 'statement' that the required result has been achieved
(c)	
B1:	"less energy lost" or equivalent